NAG Toolbox for MATLAB

g01eb

1 Purpose

g01eb returns the lower tail, upper tail or two tail probability for the Student's t-distribution with real degrees of freedom, via the function name.

2 Syntax

```
[result, ifail] = g01eb(tail, t, df)
```

3 Description

The lower tail probability for the Student's t-distribution with ν degrees of freedom, $P(\mathbf{t} \leq t : \nu)$ is defined by:

$$P(T \leq t : \nu) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\pi\nu}\Gamma(\nu/2)} \int_{-\infty}^{t} \left[1 + \frac{T^2}{\nu}\right]^{-(\nu+1)/2} dT, \qquad \nu \geq 1.$$

Computationally, there are two situations:

(a) when $\nu < 20$, a transformation of the beta distribution, $P_\beta(B \leq \beta : a, b)$ is used;

$$P(T \leq t : \nu) = \tfrac{1}{2} P_\beta\left(B \leq \frac{\nu}{\nu + t^2} : \nu/2, \tfrac{1}{2}\right) \qquad \text{when} \qquad t < 0.0$$

or

$$P(T \leq t : \nu) = \tfrac{1}{2} + \tfrac{1}{2} P_\beta\left(B \geq \frac{\nu}{\nu + t^2} : \nu/2, \tfrac{1}{2}\right) \qquad \text{when } t > 0.0$$

(b) when $\nu \geq 20$, an asymptotic normalizing expansion of the Cornish–Fisher type is used to evaluate the probability, see Hill 1970.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Hastings N A J and Peacock J B 1975 *Statistical Distributions* Butterworths

Hill G W 1970 Student's t-distribution *Comm. ACM* **13** 617–619

5 Parameters

5.1 Compulsory Input Parameters

1: **tail – string**

Indicates which tail the returned probability should represent.

tail = 'U'

The upper tail probability is returned, i.e., $P(\mathbf{t} \geq t : \nu)$.

tail = 'S'

The two tail (significance level) probability is returned, i.e., $P(\mathbf{t} \geq |t| : \nu) + P(\mathbf{t} \leq -|t| : \nu)$.

tail = 'C'

The two tail (confidence interval) probability is returned, i.e., $P(\mathbf{t} \leq |t| : \nu) - P(\mathbf{t} \leq -|t| : \nu)$.

tail = 'L'

The lower tail probability is returned, i.e., $P(\mathbf{t} \leq t : \nu)$.

Constraint: **tail** = 'U', 'S', 'C' or 'L'.

2: **t – double scalar**

t, the value of the Student's t variate.

3: **df – double scalar**

ν, the degrees of freedom of the Student's t-distribution.

Constraint: **df** ≥ 1.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

If **ifail** $\neq 0$, then g01eb returns 0.0.

ifail = 1

On entry, **tail** $\neq$ 'U', 'S', 'C' or 'L'.

ifail = 2

On entry, **df** < 1.

7 Accuracy

The computed probability should be accurate to five significant places for reasonable probabilities but there will be some loss of accuracy for very low probabilities (less than 10^{-10}), see Hastings and Peacock 1975.

8 Further Comments

The probabilities could also be obtained by using the appropriate transformation to a Beta distribution (see Abramowitz and Stegun 1972) and using g01ee. g01eb allows you to set the required accuracy.

9 Example

```
tail = 'L';
t = 0.85;
df = 20;
[result, ifail] = g01eb(tail, t, df)
```

```
result =
    0.7973
ifail =
           0
```